A. Slug Test Review and Theory

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Outline

1. What is a slug test?
2. How are slug tests performed?
3. Slug test responses
4. Theory and models of overdamped responses in low K formations
5. Theory and models of underdamped responses in high K formations
6. Examples
7. References
What is a Slug Test?

• One well field test for determining hydraulic conductivity (K)
• Entails:
  – “Instantaneously” adding or removing a slug of water in a well and monitoring water level recovery (head vs. time)
  – Substitute water level vs. time data and well geometry parameters into a mathematical model to solve for K
Why Determine K?

- Recall Darcy’s Law
  - \[ Q = KA \frac{dh}{dl} \]
  - \[ \frac{Q}{An} = v = \left( \frac{K}{n} \right) \frac{dh}{dl} \]
- **K Influences:**
  - Magnitude and direction of ground water flow and contaminant migration
  - Whether a formation is an aquifer
  - Magnitude and rate of contaminant attenuation processes (e.g., DO flux impact on MNA)
  - Risk
  - Remediation Method (e.g., pumping or excavation?)
  - Investigation and remediation costs
## K Range in Nature

<table>
<thead>
<tr>
<th>K (cm/s)</th>
<th>K (ft/day)</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;10^{-7}</td>
<td>&lt;2.8e-4</td>
<td>Crystalline rocks, clays</td>
</tr>
<tr>
<td>10^{-4} – 10^{-6}</td>
<td>0.28 – 2.8e-3</td>
<td>Clay, fine sand, poorly sorted (well graded) material, till</td>
</tr>
<tr>
<td>10^{-1} – 10^{-3}</td>
<td>283 – 2.83</td>
<td>Sand, gravel, well sorted, (poorly graded) material</td>
</tr>
<tr>
<td>&gt;10^{-1}</td>
<td>&gt;283</td>
<td>Coarse gravels, cobbles, very clean sand (no fines), karst limestone</td>
</tr>
</tbody>
</table>

Approximately 12 orders of magnitude variation in K
Alternative Ways of Determining K

- **Laboratory test**
- **Empirical relations** based on grain size analysis
- **Single or multiple well pump test**
Slug Test Advantages

- One well field test
- Easy to perform
- Minimal time and equipment
- No or minimal water for disposal, if contaminated
- Most popular test performed at contamination sites
- Developed for Low K formations \( (K < 10^{-4} \text{ cm/s}) \)
- High frequency pressure transducers and pneumatic slug testing \( > 10^{-2} \text{ cm/s} \)
How are Slug Tests Performed?

– Slug of water in
– Slug of water out
– Displacement (slug out)
– Pneumatic (slug out or slug in)
Slug In

Bucket Or Pump

Flow out

Monitor water level fall with water level sounder and time with a stopwatch if K is low, or use pressure transducer.

Use background water if use water level sounder, or distilled water with pressure transducer.

Main advantages
1. Deep well input rapid
2. Prevent water level from lowering into screen section

Modified after McCall, Geoprobe System, Inc.
Slug Out

Main advantages
1. No water use
2. Well controlled and instantaneous start

Modified after McCall, Geoprobe System, Inc.
Displacement (Mandrel) Slug Out

lower solid rod

wait for water level to recover

pull solid rod

static

Main advantages
1. No water to dispose
2. Very reproducible

Flow out

Flow in

monitor water level rise with time

Modified after McCall, Geoprobe System, Inc.
Pneumatic Slug Out

Air Pressure Gage

Valve Open

Valve Closed

Valve Closed

Valve Open

Air In regulated at constant P = 1 ft

Pressure Transducer

Water Level Falls

Equilibrium

Water Level Rises

Flow out

Flow in

Modified after McCall, Geoprobe System, Inc.
Pneumatic Slug In

Air out regulated at constant $P = 1$ ft

Modified after McCall, Geoprobe System, Inc.
Original UCONN Modified Geoprobe System for Slug-In

- Added isolation valve
- Added vacuum gauge
- Added isolation and regulator valves
- Added vacuum/pressure pump instead of foot pump

Details in Bartlett et al.
Newer Version of the UCONN Pneumatic Slug Test System

Quick Connects for P/Vac

Cable Connector

PVC
Pneumatic Test Pros and Cons

• Pros
  – High K > 10-3 cm/s (test lasts only seconds)
  – Alternative to pumping test
  – Use it to profile
  – No water used
  – “Instantaneous” input
  – Very reproducible
  – Can readily do multiple tests and get statistics
  – Equipment can be used for low K and constant head pumping tests

• Cons
  – Cost (keep in mind cost for pumping test)
  – Requires higher level of training
  – Low K formations you have to wait for equilibration
  – Screen must be submerged
Water Level Responses During Recovery

Overdamped Response

Low $K < 10^{-3}$ cm/s exponential response ($H$ or $Wd$ vs time)
Plot log Head or normalized Head [(H(Wd) = H(t)/$H_0$] vs time = linear
Note in late time log plot becomes non-linear owing to drawdown in the Formation

$H = \text{driving head = drawdown} = \text{abs} [\text{DTW}(t) - \text{DTW}(\text{static})]$
$H_0 = \text{initial head}$
$H(Wd) = Wd = \text{normalized head} = H(t)/H_0$
Overdamped Model Derivation
(Quasi Steady State)

1. In the casing at any time $t$

   \[ Q_{in} = \frac{dV}{dt} = \pi r_c^2 \frac{dh}{dt} \]

2. Flow out of the formation into the well follows a steady state form of Darcy’s Law

   \[ Q_{out} = \frac{dV}{dt} = -KSh \]

   $S =$ shape factor (depends on intake geometry)

3. Equate 1 and 2 and rearrange:

   \[ \frac{dh}{h} = -\left(\frac{K}{\pi r_c^2}\right)Sdt \]

4. Integrate 3 from $h_o$ to $h$ and $t_o = 0$ to $t$

   \[ \ln \left(\frac{h}{h_o}\right) = -\left(\frac{K}{\pi r_c^2}\right)St \]

5. Rearrange, convert $\ln$ to $\log$

   \[ \log(h) = -\left(\frac{K}{2.303\pi r_c^2}\right)St - \log(h_o) \]

   \[ y = -\quad m \quad x + b \]

   $K = \text{slope} \times \frac{2.303\pi r_c^2}{S}$
Overdamped Models

Use quasi-steady state forms of Darcy's law that depends on intake geometry (fully vs partially penetrating)

**HALF ELLIPSOID**
Dachler (1936)

\[
Q = \frac{2\pi LKH}{2.303\log\left[\frac{L}{R} + \sqrt{1 + \left(\frac{L}{R}\right)^2}\right]}
\]

**FULL ELLIPSOID**
Hvorslev (1951)

\[
Q = \frac{2\pi LKH}{2.303\log\left[\frac{L}{D} + \sqrt{1 + \left(\frac{L}{D}\right)^2}\right]}
\]

**RADIAL FLOW**
Muskat (1937)

\[
Q = \frac{2\pi LKH}{2.303\log[R_e/R]}
\]

Where:
- \( Q \) = steady state flow rate
- \( L \) = intake length
- \( K \) = hydraulic Conductivity
- \( H \) = steady state drawdown
- \( R \) = intake radius
- \( D \) = intake diameter
- \( R_e \) = radius of influence
Hvorslev Equations

**Half Ellipsoid**

**Steady State Model**

\[
Q = \frac{2\pi L K h}{2.303 \log \left[ \frac{2L}{D} + \sqrt{1 + \left( \frac{2L}{D} \right)^2} \right]}
\]

** Slug Test Equation**

\[
\log(h) = -\frac{8L K t}{5.304 d^2 \log \left[ \frac{2L}{D} + \sqrt{1 + \left( \frac{2L}{D} \right)^2} \right]} + \log(h_0)
\]

**Slope of log(h) vs. t curve**

\[
slope = \frac{8L K}{5.304 d^2 \log \left[ \frac{2L}{D} + \sqrt{1 + \left( \frac{2L}{D} \right)^2} \right]}
\]

**K equation**

\[
slope = \frac{5.304 d^2 \log \left[ \frac{2L}{D} + \sqrt{1 + \left( \frac{2L}{D} \right)^2} \right]}{8L K}
\]

\[
K = \frac{8L}{5.304 d^2 \log \left[ \frac{2L}{D} + \sqrt{1 + \left( \frac{2L}{D} \right)^2} \right]}
\]
Hvorslev Equations

**Full Ellipsoid**

**Steady State Model**

\[ Q = \frac{2\pi L K h}{2.303 \log \left[ \frac{L}{D} + \sqrt{1 + \left( \frac{L}{D} \right)^2} \right]} \]

**Slug Test Equation**

\[ \log(h) = -\frac{8L K t}{5.304 d^3 \log \left[ \frac{L}{D} + \sqrt{1 + \left( \frac{L}{D} \right)^2} \right]} + \log(h_0) \]

**Slope of \( \log(h) \) vs. \( t \) curve**

\[ \text{slope} = -\frac{8L K}{5.304 d^3 \log \left[ \frac{L}{D} + \sqrt{1 + \left( \frac{L}{D} \right)^2} \right]} \]

**K equation**

\[ K = -\frac{\text{slope} 5.304 d^3 \log \left[ \frac{L}{D} + \sqrt{1 + \left( \frac{L}{D} \right)^2} \right]}{8L} \]
Bouwer and Rice Equations

Steady State Model

\[ Q = \frac{2 \pi L K h}{2.303 \log \left( \frac{R_e}{R} \right)} \]

Slug Test Equation

\[ \log(h) = -\frac{8 L K t}{5.304 d^2 \log \left( \frac{R_e}{R} \right)} + \log(h_o) \]

Slope of \( \log(h) \) vs. \( t \) curve

\[ \text{slope} = -\frac{8 L K}{5.304 d^2 \log \left( \frac{R_e}{R} \right)} \]

K equation

\[ K = -\frac{\text{slope} 5.304 d^2 \log \left( \frac{R_e}{R} \right)}{8 L} \]

How to find \( Re \)?

- Guess?
- 50 x hole radius
- 200 x hole radius (NAVY)
- Bouwer and Rice formulas based on electric analog modeling
- Note \( Re \) in log term (so 100 x difference = 2*K)
Bouwer and Rice Method for Determining Re

\[ \ln \frac{R_e}{r_w} = \left[ \frac{1.1}{\ln(L_w/r_w)} + \frac{A + B \ln[(H - L_w)/r_w]}{L_e/r_w} \right]^{-1} \] (4) \hspace{2cm} L_w < H

\[ \ln \frac{R_e}{r_w} = \left[ \frac{1.1}{\ln(L_w/r_w)} + \frac{C}{L_e/r_w} \right]^{-1} \] (5) \hspace{2cm} L_w = H

Determine A, B, C graphically or use a polynomial regression expression (see Butler 1998)

From Bouwer, 1989, Ground Water, v. 27, n. 3, p. 304-309
Model vs. Reality

- Model predicts log-linear recovery
  - Assumes water table stays at initial level
  - In reality, in late time cone of depression forms and slows recovery

- In analyzing data only use data points that are log-linear
  - low K, use $h > 15\%h_0$
  - high K, use $h > 30\%h_0$

- Best to plot all data then delete late time non-linear points
Water Level Responses During Recovery

Critically Damped

High K formations \( K > 10^{-3} \text{ cm/s} \)
Rapid return to static but some small oscillation
How to Tell If Critically Damped

Plot log of H vs. time
If straight line = overdamped
If concave downward = critically damped
Water Level Responses During Recovery

Underdamped

High K formations $K > \text{about } 5 \times 10^{-3} \text{ cm/s}$
Rapid return to static but oscillatory
What Causes Oscillation in High K Wells?

- Inertia is key factor in causing oscillation

  Inertia is the resistance of an object to a change in its state of motion.
  Newton's first law: An object in motion will stay in motion, unless acted upon by an outside force

- Automobile analogy
  - Car accelerates: driver resists motion and is pressed to seat (driver moves slower than car)
  - Car constant speed: driver and car moving at same speed
  - Car suddenly decelerates: driver thrown forward due to going faster than car
What Causes Oscillation in High K Wells?

• Water oscillates in and out of well

• Causes mounding and depression in water table outside the well

• Water moving at “high” velocity and has inertia and overshoots static
Underdamped Response Theory
Damped Spring-Mass System

• A spring develops a restoring force proportional to how far it is stretched (and acting in the opposite direction to the stretch)
  \[ F_{\text{spring}} = -k \times x \]  
  (Hook’s Law)
  \[ \text{where: } k = \text{ spring constant} \]
  \[ x = \text{ location of the spring, at rest } x = 0. \]

• In addition, there is a damping (friction) force that resists the motion that is proportional to the velocity (v)
  \[ F_{\text{damping}} = -b \times v \]
  \[ \text{where: } v = \text{ velocity} \]
  \[ b = \text{ friction coeff, (viscous damping coeff.)} \]

• \[ F_{\text{total}} = F_{\text{damping}} + F_{\text{spring}} = -b \times v - k \times x \]
• \[ F = ma = -b \times v - k \times x \]
• \[ m \frac{dv}{dt} = -b \times v - k \times x \] (conservation of momentum (mv))
• \[ m \frac{d^2x}{dt^2} = -b \frac{dx}{dt} - k \times x \]
• Divide by m: \[ \frac{d^2x}{dt^2} = -\frac{b}{m} \frac{dx}{dt} - \frac{k}{m} x \]
• \[ \frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \left(\frac{k}{m}\right)x = 0 \]
• Define \( \omega_0 = \text{ undamped angular frequency} = \left(\frac{k}{m}\right)^{\frac{1}{2}} \)
  \[ C_d = \text{ damping ratio } = b/2m\omega_0 \]

• \[ \frac{d^2x}{dt^2} + 2C_d \omega_0 \frac{dx}{dt} + \omega_0^2 x = 0 \]
• \( C_d > 1 \) overdamped, spring returns to equilibrium without oscillating
• \( C_d = 1 \) critically damped, spring returns quickly to equilibrium without oscillating
• \( C_d < 1 \) underdamped, spring returns to equilibrium but oscillates
Underdamped Slug Test Model Solutions

- Solve conservation of momentum (differential equation) using physics of damped spring

\[ \frac{d^2 w_d}{dt_d^2} + C_d \frac{dw_d}{dt_d} + w_d = 0 \]

Where \( w_d \) = dimensionless displacement = \( w/H_0 \)
\( w \) = displacement from static
\( h_0 \) = initial displacement from static at \( t = 0 \)
\( t_d \) = dimensionless time = \( (g/L_e)^{0.5} t \)
\( h_d \) = dimensionless head = \( h/h_0 \)

- \( C_d \) = dimensionless damping parameter (fitting parameter)

- \( C_d \) = function of \( K \), pipe friction, well geometry parameters \( (r_c, r_s, L, L_e) \), fully or partially penetrating well screen

- \( L_e \) = effective length of the water column above screen (fitting parameter)
Underdamped Slug Test Model Solutions

How is K determined?

- Fit head vs time data to model to solve for $C_d$
- Methods to solve for $C_d$
  - Curve Matching, match to type curves
  - Least squares (minimize sum of the square differences), e.g., AQTESOLV, other software
  - Develop own spreadsheets use curve matching or least squares
    - Solve numerical model
- Solve for K given $C_d$
Curve Matching
Butler and Garnett

Use these equations to generate type curves to match your data and to determine $C_D$, $t_d^*$, and $t^*$

$$w_d(t_d) = e^{-\frac{C_D t_d}{2}} \left[ \cos(\omega_d t_d) + \frac{C_D}{2\omega_d} \sin(\omega_d t_d) \right], \quad C_D < 2 \quad (1)$$

$$w_d(t_d) = e^{-t_d} (1 + t_d), \quad C_D = 2 \quad (2)$$

$$w_d(t_d) = -\left( \frac{1}{\omega_d^+ - \omega_d^-} \right) \left[ \omega_d^- e^{\omega_d^+ t_d} - \omega_d^+ e^{\omega_d^- t_d} \right], \quad C_D > 2 \quad (3)$$

where
$C_D =$ dimensionless damping parameter;
g = gravitational acceleration;
$H_0 =$ change in water level initiating a slug test (initial displacement);
$L_e =$ effective length of water column in well;
$t_d =$ dimensionless time parameter, $(g/L_e)^{1/2} t$;
t = time;
w = deviation of water level from static level in well;
$w_d =$ normalized water-level deviation $(w/H_0)$;
$\omega_d =$ dimensionless frequency parameter $(=|1-(C_D/2)^2|^{1/2})$;
$\omega_d^\pm = -\frac{C_D}{2} \pm \omega_d$;

$t_d^* =$ pick time
$t^* =$ time corresponds to $t_d^*$
Steps- first normalize data to baseline P, then normalize to initial head drop (Ho), then match.
The dimensionless time axis is expanded or contracted until a reasonable match is obtained between a $C_D$ type curve and the test data. Determine $C_D$, $t_d^*$, and $t^*$.
4) The radial hydraulic conductivity ($K_r$) is estimated by substituting the well-construction parameters, the $C_D$ value, and the match-point ratio ($t_d^*/t^*$) into the appropriate equation:

**Unconfined--High-K Bouwer and Rice Model (Springer and Gelhar, 1991)**

$$K_r = \frac{t_d^* r_c^2 \ln[R_e/r_w]}{t^* 2bC_D}$$  \hspace{1cm} (4)

**Confined--High-K Hvorslev Model (Butler, 1997)**

$$K_r = \frac{t_d^* r_c^2 \ln[b/(2r_w) + (1 + (b/(2r_w))^2)^{0.5}]}{2bC_D}$$  \hspace{1cm} (5)

where

$b =$ screen length;
$R_e =$ effective radius parameter of Bouwer and Rice (1976);
$r_c =$ effective radius of well casing (corrected for radius of transducer cable);
$r_w =$ radius of well screen or borehole.

$$L_e = \left(\frac{t^*}{t_d^*}\right)^2 g$$

where

$C_D =$ dimensionless damping parameter;
$g =$ gravitational acceleration;
$H_0 =$ change in water level initiating a slug test (initial displacement);
$L_e =$ effective length of water column in well;
$t_d =$ dimensionless time parameter, $(g/L_e)^{1/2} t$;
$t =$ time;
$w =$ deviation of water level from static level in well;
$w_d =$ normalized water-level deviation ($w/H_0$);
$\omega_d =$ dimensionless frequency parameter $(=|1-(C_D/2)^2|^{1/2})$;
$\omega_d^* = -\frac{C_D}{2} \pm \omega_d$;
Oscillatory Slug Test Model Differences

- Hvorslev (partial penetrating) vs. Bouwer and Rice (fully penetrating)
- Confined or unconfined
- Apply to all responses or just oscillatory (low values of $C_d$)
- Corrects for pipe friction or not (added term within $C_d$)
- Corrects for depth of pressure transducer
- Corrects for well skins
Issues In Conducting and Analyzing Underdamped Slug Tests

• Test lasts only a few seconds
  – Need p-transducer that responses rapidly (< 1 sec)

• Hard to make ho instantaneous
  – Can use time of peak or trough for start time

• P-transducer should be located near water surface
  – Most models based on measuring the difference in water level from static
  – Pressure measurement are affected by the acceleration of the water column and is a function of the depth of placement of the pressure transducer
  – Best to keep p-trans depth close to static if not it leads to under estimation of K by a factor of 2 or more.
  – Butler suggests put p-trans less than 1.5 ft below static avoids the need for correction. Test by observing maximum normalized H (h/ho), if less than 0.9 move p-trans closer to static.

• Have to handle a lot of data to isolate test
  – E.g, 10 Hz, takes 2 minutes for set up = 1200 rows of data for a spreadsheet

• Are model assumptions met?

• Friction appears only important for small diameter wells
  – Butler (2002) found wells r > 0.012 m (1.2 cm) or diameter less than 1 inch, no correction is needed
Example Test

Three tests
27590 lines of data on spreadsheet
Test Example

P-trans within 1.5 feet of water surface
$R_c = 1''$

<table>
<thead>
<tr>
<th></th>
<th>K (cm/s)</th>
<th>K(ft/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>test 1</td>
<td>0.1046</td>
<td>297</td>
</tr>
<tr>
<td>test2</td>
<td>0.1039</td>
<td>295</td>
</tr>
<tr>
<td>test 3</td>
<td>0.1041</td>
<td>295</td>
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<tr>
<td>average</td>
<td>0.1042</td>
<td>295.37</td>
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<tr>
<td>Std Dev</td>
<td>0.000361</td>
<td>1.022046</td>
</tr>
<tr>
<td>%RSD</td>
<td>0.35</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Select References

• Chen, C., and C. Wu, 2006, Analysis of Depth-Dependent Pressure Head of Slug Tests in Highly Permeable Aquifers, *Ground Water*, 44 (3) :472-477